MAT 090 – Brian Killough's Instructor Notes
Strayer University

Success in online courses requires self-motivation and discipline. It is anticipated that students will read the textbook and complete sample exercises as required, before working on the My-Math-Lab (MML) modules. In general, our textbook is a reference and all of our homework and tests are completed within the MML modules. A summary of the weekly modules and corresponding textbook material is in the table below.

Module	Textbook Material
1	Sections 1.1 to 1.9, 2.1 to 2.5
2	Sections 2.6 to 2.9
3	Sections 3.1 to 3.8
4	Sections 4.1 to 4.4
5	Sections 4.5 to 4.7
6	Sections 5.1 to 5.5
7	Sections 6.1 to 6.6
8	Sections 7.1 to 7.5
9	Sections 8.1 to 8.5
10	Sections 9.1 to 9.3

These instructor notes are intended to supplement the course by providing a short summary of the key material and providing solutions to example problems that emphasize important or confusing topics. I hope you find them useful.

MML Module #1

Chapter 1 Whole Numbers

We will not cover sections 1.1 to 1.6 in this course. It is assumed that this material is review, but many of you will need to quickly review this content before moving on to Section 1.7. In addition, the operations with whole numbers and integers can be done with a common calculator.

It is important to know the various types of mathematical numbers. This is best done by describing the categories of numbers and giving some examples.

Name	Description	Examples
Natural Numbers	Counting Numbers	1,2,3,4,5 …
Whole Numbers	Counting Numbers + ZERO	0,1,2,3,4,5 …
Integers	Number Line (negatives + positives)	…-3,-2,-1,0,1,2,3 …

Solving Equations ... In this Chapter, the solution of equations does not involve any complex manipulation of numbers or variables, but you will need to perform simple operations of addition, subtraction, multiplication or division.

Example X = 2 + 12 .... x = 14

Example X = 110 - 30 .... x = 80

IMPORTANT: The rule to remember in this chapter is whatever mathematical process you do on one side of an equation (add, subtract, multiply, divide), you must perform the same process on the other side of the equation. You may add or subtract numbers from both sides of an equation to get a variable by itself. This is called isolating the variable and is essential to solving equations. By isolation … we mean, getting the variable on to one side of the equation and the numbers on the opposite side.

Example $3 = x + 5$ (subtract 5 from both sides) >> $3 - 5 = x + 5 - 5$
$-2 = x$ therefore, $x = -2$

Any solution to an equation can be easily verified using substitution. **You should get into a habit of taking the time to check your solutions.** Replace your answer back into the ORIGINAL equation.

Example 10 = x + 5 (subtract 5 from both sides)
10 - 5 = x + 5 - 5
5 = x
Check .... 10 = 5 + 5, 10 = 10 ... looks good !!

Example 4 * x = 16 (divide both sides by 4)
4 * x / 4 = 16 / 4
x = 4

Notice in the equation above we use a DOT for multiplication. Sometimes you will see a "x" and other times you will see a DOT. Either one is multiplication.

Exponential Notation

These are often called “powers” and represent a simplified method to write many multiplications.

Example $x \cdot x \cdot x \cdot x \cdot x = x^5$ where “x” is the base and “5” is the power.

Example $-7^4 = -(7)(7)(7)(7) = -2401$

Note: DO NOT multiply the base and the power but think of expanding the exponent.

Example $(1+2)^3 = (3)^3 \neq 9$ but $(3)^3 = 3 \cdot 3 \cdot 3 = 27$

Order of Operations

You may remember the saying “Please Excuse My Dear Aunt Sally” which uses the first letter of each word to represent the order of operations .. that is … **PEMDAS**

Parenthesis – Exponents – Multiplication and Division – Addition and Subtraction

When simplifying expressions we must use these rules in the proper order:

(1) Work from the inside out, if there are multiple sets of parenthesis
(2) Evaluate parenthesis … also consider **absolute value** symbols as well
(3) Evaluate exponents (left to right)
(4) Perform multiplication and division (left to right)
(5) Perform addition and subtractions (left to right)

NOTE: Always go left to right if you have several of the same type of operations (multiplication and division, or addition and subtraction).

Now lets try a few examples … follow them closely

Example $5(-3)^2 - 2(-4)^2 = 5(-3)(-3) - 2(-4)(-4) = 5(9) - 2(16) = 45 - 32 = 13$

Example $-2\left[(3-4)^3 - 5\right] + 7 = -2\left[(-1)^3 - 5\right] + 7 = -2\left[-1-5\right] + 7 = -2\left[-6\right] + 7 = 12 + 7 = 19$

EXAMPLE 10 Simplify: $4^2 \div (10 - 9 + 1)^3 \cdot 3 - 5$.

$4^2 \div (10 - 9 + 1)^3 \cdot 3 - 5$

$= 4^2 \div (1 + 1)^3 \cdot 3 - 5$ Subtracting inside parentheses

$= 4^2 \div 2^3 \cdot 3 - 5$ Adding inside parentheses

$= 16 \div 8 \cdot 3 - 5$ Evaluating exponential expressions

$= 2 \cdot 3 - 5$

$= 6 - 5$ Doing all multiplications and divisions in order from left to right

$= 1$ Subtracting

Chapter 2 Introduction to Integers - Sections 2.1 to 2.5

Absolute Value: Absolute value is merely the POSITIVE equivalent of any number: $|-15| = 15$

Example $6|7 - 4\cdot 3| = 6|7 - 12| = 6|-5| = 6(5) = 30$

Addition, Subtraction, Multiplication and Division - Whole Numbers and Integers

Identity Property of Addition – achieve an "identical" term Example: a+0=a
Inverse Property of Addition – uses the inverse term to simplify Example: a + (-a) = 0

Example 3 + (-2) = 3 – 2 = 1

Example -4 + (-6) = -4 - 6 = -10

Example 10 + (-3) + (-8) = 10 – 3 – 8 = 10 -11 = -1

Addition and subtraction is most difficult when there are different signs. We can always think of subtraction as just adding a "negative". Also, remember that subtracting a "negative" is the same as adding since the two negative signs will cancel each other.

Example: A – B = A + (- B)

Example: A – (- B) = A + B

Example 5 – (-17) = 5 + 17 = 22

Example -6 – (-3) + 8 -11 = -6 + 3 + 8 -11 = -3+8-11 = 5-11 = -6

NOTE: Some calculators (TI-83) have a "-" sign for arithmetic (subtraction) and a "(-)" sign for making a number negative. BE CAREFUL when entering negative numbers on your calculator.

Identity Property of Multiplication – achieve an "identical" term Example: $a \cdot 1 = a$

Multiplication Property of Zero – always achieve zero when multiplying by zero Example: $a \cdot 0 = 0$

Remember the **product of two negative numbers is a positive number**. Think of the signs canceling.

Example (-11)(-1) = 11

Some books use an "X" or a "DOT" for multiplication symbols. If there is no symbol, it is implied that we multiply.

Example (10)(-7) = -70

Example $6 \div 0 = \frac{6}{0} = \mathit{Undefined}$ **NEVER divide by ZERO … that is undefined.**

Note: You will find that the book uses 3 different ways to express a DIVISION. Above we see two example .... the division symbol and "6 over 0". It is also possible to show a division using a "SLASH", such as 6/0. All of these are division operations.

Example $0 \div (-10) = 0$ This one is OK … since we are dividing by -10

MML Module #2

Chapter 2 Algebraic Expressions - Sections 2.6 to 2.9

Algebraic Expressions

It is possible to convert standard English expressions into **Algebraic Expressions** using variables (such as X,Y,Z) and numbers. For example,

The sum of a number and 6 ..... x + 6
Five times a number decreased by 3 .... 5x - 3

Evaluating Expressions requires that we substitute numbers into variable expressions. For example, if we want to evaluate the algebra expression 5(x+3) for x = 5, then we replace 5 for the variable "x".

5(x+3) .... 5(5+3) = 5(8) = 5*8 = 40

Distributive Law ... we often use parentheses to separate terms in an equation. If we want to expand those terms, we need to DISTRIBUTE across the terms. For example ....

a(b+c) = ab + ac .... notice it DOES NOT equal "ab+c"

Example 2(x+3) = 2x + 2(3) = 2x + 6

Solving Equations

Now, lets consider equations that have a number in front of the variable. To get rid of that number and solve for the variable, we must divide BOTH sides of the equation by that number. For example:

Example -4y = 32 … to solve for Y, we must divide BOTH sides by -4 to get: Y = -8

Just remember that dividing by 4 is the same as multiplying by 1/4. This is often easier to use. Finally, we may multiply by (-1) to get a sign change in a problem. All of these rules are needed to solve for variables.

Example

$-13x + 7 = -19$ subtract 7 from both sides

$-13x = -26$ divide both sides by -13 to sove for x

$x = 2$

When working with equations you can move a term across the equal sign, **but you must change its sign**. In the previous problem if you moved the 7 to other side it would become –7 and then could be combined with –19 to get –26.

Example $-5x = -20$ (divide both sides by -5) ... then ... $\frac{-5x}{-5} = \frac{-20}{-5}$ ... and ... $x = \frac{-20}{-5} = 4$

Example $-m = -17$ (multiply by -1 on both sides to change the sign) ... then ...
$-m(-1) = -17(-1)$ therefore $m = 17$

Example -3y – 2 = -5 – 4y
-3y + 4y = -5 + 2
y = -3

Adding and Subtracting Polynomials

NOTE: Section 2.9 in the book does not exist in the MML online textbook. They use Section 10.1, which is the same material.

Know the pieces of a polynomial by placing highest powers first (on the left) and the lower successive powers to the right.

$2x^3 + 3x^2 - 2x - 7$

Order or Power = 3 (highest power term)
Leading Coefficient = 2 (constant in front of the highest power term)
Constant Term = -7 (last term with no variable, including the sign)

When adding or subtracting polynomials we can only add terms that are similar (same powers).

Example $\left(2y^2 - 3y - 8\right) + \left(y^2 + 4y - 1\right) = 3y^2 + y - 9$

Example $(5x - 3) + \left(x^3 + 3x - 2\right) + (-2x - 3) = x^3 + 6x - 8$

Example (y-2) – (7y-9) = y-2-7y+9 = -6y+7

Example $\left(2n^2 - n^7 - 6\right) - \left(2n^3 - n^7 - 8\right) = 2n^2 - n^7 - 6 - 2n^3 + n^7 + 8$
$-2n^3 + 2n^2 + 2$

MML Module #3

Chapter 3 Fraction Notation: Multiplication and Division

Divisibility and **Factorizations** may be new terms for many of you. We use these to deal with larger numbers in fractions and to reduce fractions into simple terms. So, lets look at each.

Divisibility ... a number is divisible (or dividable) by another number if the remainder is ZERO. So, basically we want the division to produce a whole number answer.

20 is divisible by 2, since 20 / 2 = 10

20 is NOT divisible by 3, since 20 / 3 = 6.6666 .. NOT a whole number

Factorization ... if a number is divisible by another number, then we know that it is possible to "factor" the number into smaller pieces. We call this process, factorization. So, lets look at this example ....

20 ... since it is divisible by 2, we know that 20 = 2 * 10
BUT, 10 can be broken into 2*5 = 10
SO, the entire factorization of 20 into its smallest parts is .... 20 = 2 * 2 * 5

This is the process of <u>FACTORIZATION</u> ... lets try another

Factor 36 = 2 * 18 = 2 * 3 * 6 = 2 * 2 * 3 * 3

NOTE ... Prime numbers are not divisible by anything and DO NOT have any factors. For example, prime numbers are 2,3,5,7,11,13,17,19, etc.

Now lets talk about **simplifying fractions** … this is very important. We ALWAYS want to reduce fractions into their simplest form by simplifying them.

Example: Simplify $\frac{18}{45}$ … we can also write this as 18/45, when typing on one line.

Now, to reduce we must find the Greatest Common Factor (GCF) of the top and bottom. This is the largest common number that is part of BOTH the top and bottom. To find this, it is best to take the fraction apart … separate it, into it parts to find the factors that can be reduced. To do this, break down the fraction into the smallest multiples you can.

$\frac{18}{45} = \frac{2 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 5}$ … now, we can cancel out terms that are common on the top and bottom. So in this case, we can remove a pair of “3’s” from the top and bottom. That leaves us with 2/5 as the answer.

Multiplying Fractions: When we multiply fractions, we can multiply the tops and then the bottoms, separately. Also, remember … a whole number (like 9) can ALWAYS be written as a fraction, such as 9/1. So lets try this one …

Example: Simplify and Reduce $\frac{1}{8} * \frac{2}{3} = \frac{2}{24} = \frac{2}{2*12} = \frac{1}{12}$

Multiply the TOP terms to get 2. Then multiply the bottom terms to get 24. Now, simplify as we did in the previous example. So find the factors on the top and bottom. When we eliminate the 2 on top, it leaves a 1 behind … this is important. It DOES NOT leave a zero behind, but always a 1.

Dividing Fractions: When we divide fractions, we actually MULTIPLY. We take the division sign and change it to a multiplication. Then we must FLIP-OVER the second part to create what is called the “RECIPROCAL” of the second term. Then we multiply, like above. Lets try one …

Example $6 \div 2 = 6 \cdot \frac{1}{2} = \frac{6}{1} \cdot \frac{1}{2} = \frac{6}{2} = 3$

Example Simplify and Reduce $\frac{7}{4} \div \frac{3}{8} = \frac{7}{4} * \frac{8}{3} = \frac{56}{12} = \frac{7*4*2}{4*3} = \frac{7*2}{3} = \frac{14}{3}$

Notice … the division is converted to a multiplication. Then the second term is FLIPPED over. Then we multiply and simplify.

Reciprocals and Division: When dividing fractions we convert the division into a multiplication and flip over (called reciprocal) the following fraction. This is called “**INVERT and MULTIPLY**”

Example $\frac{1}{8} \div \frac{5}{16} = \frac{1}{8} \cdot \frac{16}{5} = \frac{16}{40} = \frac{2}{5}$ Example $\frac{3}{5} \div 3 = \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{5}$

Solving fractional equations can be difficult, but the best trick is to find the Least Common Denominator (LCD) and **multiply EVERY term in the equation by the LCD to remove all fractions in one step.** Note: Only consider the fractions when finding the Least Common Denominator (LCD). Whole numbers are really fractions with a ONE on the bottom and do not affect the selection of the LCD. Here are some examples …

Example

$-\frac{3}{7}x = 4$ the LCD is 7 since the right side is really a fraction, $\frac{4}{1}$

Multiply the entire equation by 7 to get : $\frac{7}{1}\cdot\left(\frac{-3}{7}x\right) = 4\cdot 7$ then $-3x = 28$ $x = \frac{-28}{3}$

Example

$\frac{1}{4}+\frac{1}{5}=\frac{x}{2}$ LCD=20, Multiply all terms by the LCD

$\frac{20}{4}+\frac{20}{5}=\frac{20x}{2}$ then $5+4=10x$

$9 = 10x$ divide both sides by 10 then: $x = \frac{9}{10}$

MML Module #4

Chapter 4 Fraction Notation: Addition and Subtraction - Sections 4.1 to 4.4

When adding and subtracting fractions we will need to find the **Least Common Multiple (LCM)** of the denominators (or bottom) of the fractions. The LCM is the smallest number that is a multiple of both. The LCM will be greater than or equal to the largest number. For example ....

What is the LCM of 2 and 3 ? ... the answer is 6.

What is LCM of 10 and 12 ? ..... first we list the multiples of 6 and 10
6 ... multiples are 6, 12, 18, 24, **30**, 36, 42, etc
10 ... multiples are 10, 20, **30**, 40, 50, etc
Notice that the smallest COMMON multiple or LCM is 30.

Adding and Subtracting … in order to add or subtract fractions, they MUST have the same denominators (bottom terms). If that is the case, then we can simply ADD the top parts of the fractions and the bottom terms will remain the same.

$$\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$$

$$\frac{3}{12}+\frac{5}{12}=\frac{3+5}{12}=\frac{8}{12}$$

If the denominators are NOT the same, we must convert fractions to make them the same. We use LCM to do this process, or in some cases, it is called the **Least Common Denominator (LCD)**.

Example Simplify and Reduce $\frac{3}{2}-\frac{2}{3}$ … notice the bottom numbers are different. We cannot perform this subtraction yet. The bottom terms MUST be the same. So, first we have to find the Least Common Denominator (LCD) and then convert each fraction. The LCD is the smallest common number that the denominators from each fraction can be divided into. One way to find this, is to multiply both bottom terms together … so we get 2*3 = 6. This could be used as an LCD. So lets convert each fraction into a denominator of 6.

First Term: 3/2 … to get a 6 on the bottom, we must multiply both the top and bottom by 3. We are allowed to do this, since multiplying by 3/3 is the same as multiplying by 1, and that does not change the value of the fraction. So we get:

$$\frac{3}{2} * \frac{3}{3} = \frac{9}{6}$$

Second Term: 2/3 … to get a 6 on the bottom, we must multiply both the top and bottom by 2. So we get:

$$\frac{2}{3} * \frac{2}{2} = \frac{4}{6}$$

Now that the bottoms are the same, we can perform the subtraction: $\frac{9}{6} - \frac{4}{6} = \frac{5}{6}$

When we add or subtract fractions, we only perform the operation on the top part of the fraction. The bottom denominator stays the same.

Be sure to practice these types of problems .. there are many in the book and they are very important to master !!!

Solving Fraction Equations with Addition and Subtraction

Example

$y - \frac{1}{4} = \frac{1}{2}$ (add $\frac{1}{4}$ to both sides) >>> $y - \frac{1}{4} + \frac{1}{4} = \frac{1}{2} + \frac{1}{4}$

$y + 0 = \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ therefore, $y = \frac{3}{4}$

Example

$r + \frac{3}{5} = -\frac{7}{10}$ … move the 3/5 to the other side to get -3/5.

$r = -\frac{7}{10} - \frac{3}{5} = -\frac{7}{10} - \frac{6}{10} = -\frac{13}{10}$ … notice how 3/5 was converted to 6/10 to perform the subtraction using common denominators.

Example

$\frac{1}{3}s + \frac{7}{9} = \frac{4}{3}s$ (subtract $\frac{1}{3}s$ from both sides or move the term right and change its sign)

$\frac{7}{9} = \frac{4}{3}s - \frac{1}{3}s = \frac{3}{3}s = s$ ... therefore ... s = $\frac{7}{9}$

MML Module #5

Chapter 4 Mixed Numbers - Sections 4.5 to 4.7

Converting improper fractions to mixed numbers (and the reverse) is an essential process. An improper fraction is just a single fraction (has a top and bottom part) that can be converted to a whole number and some leftover parts. Here are a few examples:

Example: Convert $2\frac{7}{9}$ to an improper fraction. First, take the denominator (bottom) and multiply it times the whole number on the left (2). This gives us 18. Now add that to the top (7) to get 25 on the top. The answer is then $\frac{25}{9}$. Notice, how we keep the same denominator on the bottom, but the top is now larger. You can think of the original "2" out front as really the same as $2 = \frac{18}{9}$. When we add that to the right side (7/9) we get the end result of 25/9.

$$6\frac{2}{3} = \frac{20}{3} \qquad 6 \cdot 3 = 18; \quad 18 + 2 = 20; \quad \text{keep the denominator.}$$

$$8\frac{2}{9} = \frac{74}{9} \qquad 9 \cdot 8 = 72; \quad 72 + 2 = 74; \quad \text{keep the denominator.}$$

Adding and Subtracting Mixed Numbers ... in your book they suggest that you add or subtract the WHOLE numbers first and then add or subtract the FRACTIONS next. The second step will often require us to find an LCD to deal with the fractions. So, be sure to practice these. In the example below, notice how the first fraction was converted to a denominator of 6 so that it could be added to the second fraction of 5/6.

EXAMPLE 1 Add: $5\frac{2}{3} + 3\frac{5}{6}$. Write a mixed numeral for the answer.

The LCD is 6.

$$\begin{array}{rcr} 5\,\frac{2}{3} \cdot \frac{2}{2} & = & 5\frac{4}{6} \\ +\,3\,\frac{5}{6} & = & +\,3\frac{5}{6} \\ \hline & & 8\frac{9}{6} = 8 + \frac{9}{6} \\ & & = 8 + 1\frac{1}{2} \\ & & = 9\frac{1}{2} \end{array}$$

To find a mixed numeral for $\frac{9}{6}$, we divide:

$$\begin{array}{r} 1 \\ 6\overline{)9} \\ \underline{6} \\ 3 \end{array} \qquad \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$$

$\frac{19}{2}$ is also a correct answer, but it is not a mixed numeral, which is what we are working with in Sections 4.5, 4.6, and 4.7.

Sometimes you will have to use some clever tricks to convert whole numbers into fractions so that we can add or subtract. For example ....

EXAMPLE 5 Subtract: $13 - 9\frac{3}{8}$.

$$\begin{array}{rcr} 13 & = & 12\frac{8}{8} \\ -\;9\frac{3}{8} & = & -\;9\frac{3}{8} \\ \hline & & 3\frac{5}{8} \end{array} \qquad 13 = 12 + 1 = 12 + \frac{8}{8} = 12\frac{8}{8}$$

Multiplying and Dividing Mixed Numbers ... When doing multiplication or division with mixed numbers, it is best to convert the mixed numbers into fractions first and then do the multiplication or division.

Caution ... DO NOT multiply the WHOLE numbers and then the FRACTIONS. Convert each part to a fraction first and then multiply and simplify.

EXAMPLE 4 Multiply: $2\frac{1}{4} \cdot 3\frac{2}{5}$.

$$2\frac{1}{4} \cdot 3\frac{2}{5} = \frac{9}{4} \cdot \frac{17}{5} = \frac{153}{20} = 7\frac{13}{20}$$

CAUTION!

$2\frac{1}{4} \cdot 3\frac{2}{5} \neq 6\frac{2}{20}$. A common error is to multiply the whole numbers and then the fractions. The correct answer, $7\frac{13}{20}$, is found only after converting to fraction notation.

MML Module #6

Chapter 5 Decimal Notation

Fractions and Decimals are interchangeable. We need to know how to convert a fraction to a decimal and the reverse process. Converting a fraction to a decimal is easiest, since we can just use our calculator to do the division of top and bottom.

Example: 1/2 .... 0.5

Example: 2/3 .... 0.666666

If we want to **convert a decimal into a fraction**, that is harder. First, determine the number of digits to the RIGHT of the decimal. Next, move that decimal to the farthest point to the right and divide by a factor of 10 that is equal to the number of digits you moved to the RIGHT. For example, if I move the decimal 3 places RIGHT, then I will divide by 10^3, or 10*10*10 = 1000.

EXAMPLE 6 Write fraction notation for 0.876. Do not simplify.

0.876 $\quad$ 0.876. $\quad$ $0.876 = \frac{876}{1000}$

3 places $\quad$ 3 zeros

EXAMPLE 7 Write 56.23 as a fraction and as a mixed numeral.

To write as a fraction, we follow the procedure outlined above:

$$56.23 \qquad 56.23. \qquad 56.23 = \frac{5623}{100}$$

2 places 2 zeros

Addition and Subtraction of Decimals is quite easy with a calculator, but if you need to do it by hand, then you need to understand the process of lining up the decimals and adding or subtracting in a vertical manner. Here are some examples. In general, you should know how to do this, if required, but we will typically use a calculator.

EXAMPLE 3 Add: 3456 + 19.347 .

$$\begin{array}{rl}
\overset{1}{} & \\
3\,4\,\overset{1}{5}\,6.0\,0\,0 & \text{Writing in the decimal point and extra zeros} \\
+\quad 1\,9.3\,4\,7 & \text{Lining up the decimal points} \\
\hline
3\,4\,7\,5.3\,4\,7 & \text{Adding}
\end{array}$$

EXAMPLE 5 Subtract: 23.08 − 5.0053 .

$$\begin{array}{rl}
\overset{1}{\cancel{2}}\,\overset{13}{\cancel{3}}.0\,\overset{7}{\cancel{8}}\,\overset{9}{\cancel{0}}\,\overset{10}{\cancel{0}} & \text{Writing two extra zeros to the right of the last digit} \\
-\quad 5.0\,0\,5\,3 & \\
\hline
1\,8.0\,7\,4\,7 & \text{Subtracting}
\end{array}$$

Multiplication and Division of Decimals is also quite easy with a calculator, but if you need to do it by hand, then you need to understand the process. Here are some examples below. You should review this in more detail in the book. Overall, use your calculator to check the answers at all times.

EXAMPLE 3 Multiply: −0.14 × 0.867 .

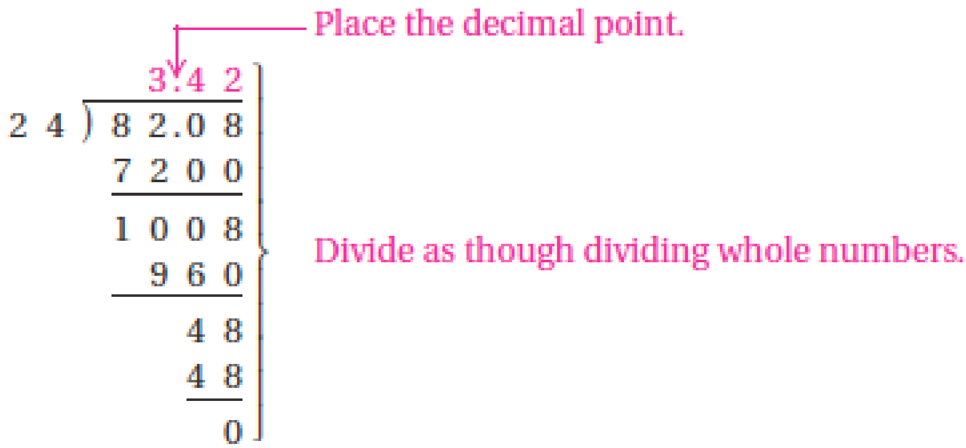
Multiplying the absolute values, we have

$$\begin{array}{rl}
0.8\,6\,7 & \text{(3 decimal places)} \\
\times\quad 0.1\,4 & \text{(2 decimal places)} \\
\hline
3\,4\,6\,8 & \\
8\,6\,7\,0 & \\
\hline
0.1\,2\,1\,3\,8 & \text{(5 decimal places)}
\end{array}$$

Since the product of a negative and a positive is negative, the answer is −0.12138 .

EXAMPLE 1 Divide: 82.08 ÷ 24.

We have

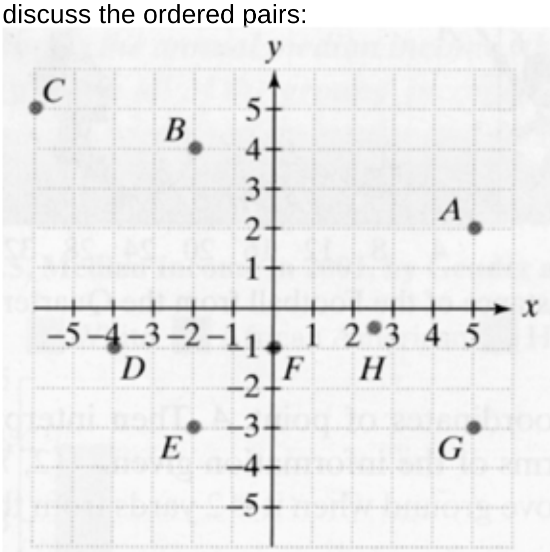


MML Module #7

Chapter 6 Introduction to Graphing and Statistics

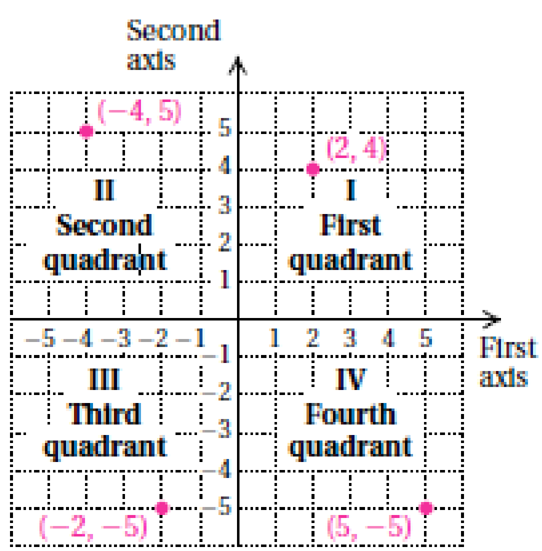
This chapter introduces tables (Section 6.1), Bar Graphs and Line Graphs (Section 6.2). Rather than show examples here, it is best to view them in the textbook and to practice reading the data. I think most students will find this material rather easy, but just PRACTICE.

The Cartesian Coordinate system is the X-Y plane where points (**ordered pairs**) can be plotted using the X and Y coordinates. The X-axis runs left-to-right and the Y-axis runs up-and-down. We plot an ordered pair (X,Y) but finding the X and Y coordinate points on the graph. So lets look at the following graph and discuss the ordered pairs:



Ordered Pairs or Points on the Graph:
A = (5,2), B = (-2,4), C = (-6,5), D = (-4,-1),
E = (-2,-3), F = (0,-1), G = (5,-3), H = (2.5, -0.5)

The axes are divided into 4 quadrants or regions.
See the figure on the right



Graphing Equations ... In this class we will graph linear equations (LINES) using tables to make a list of points (X,Y).

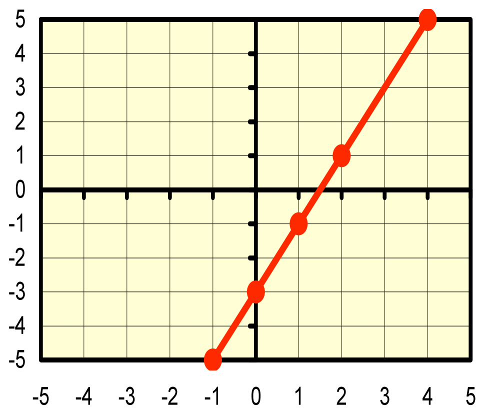
Example $\quad Y = 2X - 3$

To graph this line set up an X and Y table and choose at least 3 points (it only takes two to make a line but this makes sure there is no error in your calculations). Use the chosen X points and solve for the Y point that goes with each one. Graph the points and connect the line.

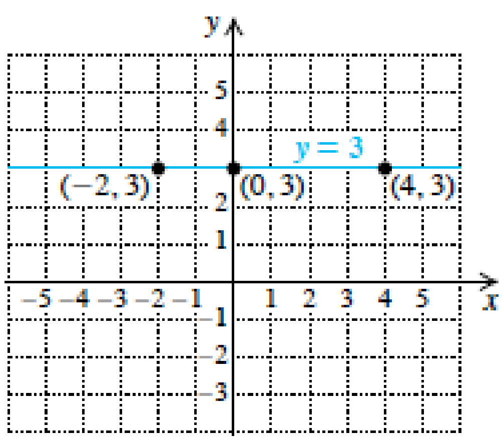
HINT: It does not matter what you choose for X, there are an infinite number of points on a LINE, so pick some reasonable values for X to give you points with a reasonable value of Y that fit on your graph.

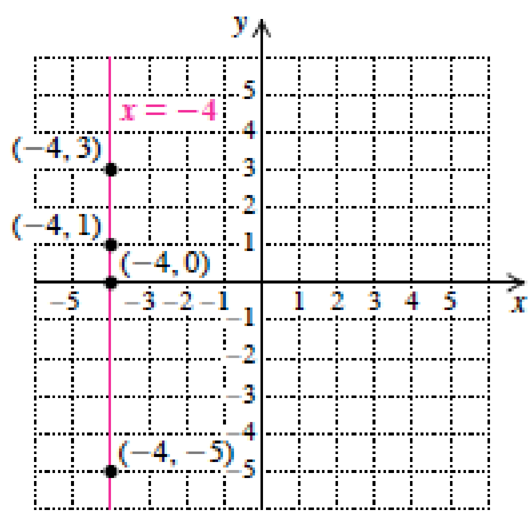
X	Y
0	- 3
1	-1
2	1
3	3

Graph: Y = 2 X - 3



In the special case where we have only ONE variable, such as X=2 or Y=3, these will be vertical or horizontal lines. So, if X=number, then we have a **VERTICAL** line. If Y=number, then we have a **HORIZONTAL** line. Here are some examples of each ... Y = 3 and X = -4.





STATISTICS are used to analyze data sets numerically. Evaluating key statistical parameters is required for many disciplines such as engineering, medicine, chemistry, etc. When examining data it is helpful to use “measures of central tendency” or averages. These are known as: **Mean, Median, and Mode**.

MEAN – The most common statistical number is the mean or common average. It is merely the sum of all the data points divided by the number of data points.

Example: 70, 66, 65, 69 $$\frac{70+66+65+69}{4} = \frac{270}{4} = 67.5.$$

MEDIAN – Another common statistical number is the median, which represents the “middle point” of the data. This requires us to list the data from smallest to largest. If the number of data points is ODD, then the median is the middle number of the data set. If the number of data points is EVEN, then the median is the average (mean) of the middle TWO terms. In the previous example, the MEDIAN is (66+69)/2 = 67.5.

Example: 8, 10, 11, 12, 13, 18 $MEDIAN = \frac{11+12}{2} = 11.5$

Example: 1,2,3,4,5 MEDIAN = 3 (middle term)

MODE – The mode is the one item of data that occurs the most often or frequently in a data set. If there are two items that occur the most, the set of data is called “bimodal”. If no item of data is repeated (as with the previous examples) then we say there is NO MODE.

Example: 6, 3, 2, 1, 2, 4, 8, 9, 2 MODE = 2

Probability is used to predict the likelihood of an event based on some known data. So, if we look at a set of data we can find the probability using the following equation.

$$P = \frac{Number\ of\ occurrences\ or\ events}{Total\ number\ of\ trials\ or\ attempts}$$

The “law of large numbers” implies that probabilities are more accurate as we increase the number of trials. Try rolling a dice or flipping a coin. The probabilities become more accurate as we do it more often.

Example Birds at a Feeder: The last 30 birds that fed at the bird feeder were 14 finches, 10 cardinals, and 6 blue jays. Determine the empirical probability that the next bird to feed is

(a) Finch P(finch) = 14/30 = 7/15 … always reduce fractions when possible

(b) Cardinal P(cardinal) = 10/30 = 1/3 ... or 0.3333

(c) Blue Jay P(blue jay) = 6/30 = 1/5 ... or 0.20

Example Grade distributions over the past 3 years for a course are shown in the chart below. If a student plans on taking a class with the same instructor, determine the empirical probability of the following grades:

(a) Grade = A, P(A) = 43/645 … where
Total trials = 645 = 43+182+260+90+62+8

(b) Grade = C, P(C) = 260/645 = 52/129 (reduced)

(c) Grade above D, P(>D) = $\frac{43+182+260+90}{645} = \frac{575}{645} = \frac{115}{129}$

Grade	Number
A	43
B	182
C	260
D	90
F	62
I	8

Example	Each letter of the word “MISSISSIPPI” is placed on a piece of paper and all 11 pieces are placed in a hat. If one letter is selected at random from the hat, find the probability that:	
Case 1	The letter “S” is not selected	Probability (not “S”) = P(~s) = 7 possible outcomes / 11 total = 7/11 = 0.636
Case 2	The letter “I” or “P” is selected	P(I or P) = P(I) + P(P) = 4/11 + 2/11 = 6/11 = 0.545
Case 3	The letter “W” is selected	P(w) = 0 … not possible.

MML Module #8

Chapter 7 Ratio and Proportion

A **ratio** is a comparison of two quantities. We use a fraction notation, or a special notation with a semi-colon. For example, the RATIO of 1 to 2 is ....

$$\frac{1}{2} \quad or \quad 1:2$$

We can also make ratios of decimals or mixed numbers, but we typically reduce the numbers to a single fraction in the end. For example .... the ratio of 2.4 to 9.2

$$\frac{2.4}{9.2} = \frac{2.4}{9.2} \cdot \frac{10}{10} = \frac{24}{92} = \frac{4 \cdot 6}{4 \cdot 23} = \frac{4}{4} \cdot \frac{6}{23} = \frac{6}{23}.$$

A **rate** is another form of a ratio between two measurements. The most common example is speed. If a person travels 145 miles in 2.5 hours, then the ratio of those quantities gives us a rate of travel, or the speed.

$$\text{The rate is } \frac{145 \text{ mi}}{2.5 \text{ hr}}, \quad \text{or} \quad 58 \text{ mph.}$$

Miles per hour is abbreviated mph.

Unit Price is the ratio of the price to the number of units. For example .... if 20 pounds of food cost $25 then the unit price is ...

$$\begin{aligned}\text{Unit price} &= \frac{\text{Price}}{\text{Number of units}} \\ &= \frac{\$25}{20 \text{ lb}} \\ &= \frac{25}{20} \cdot \frac{\text{dollars}}{\text{lb}} \\ &= 1.25 \text{ dollars per pound, or } \$1.25/\text{lb}\end{aligned}$$

When two ratios are equal, we call them **proportional**. We can check if two ratios are proportional using cross-multiplication. Basically, we multiply the numbers ACROSS the equal sign to see if they are equal. For example, is 1/2 proportional to 3/6 ? Yes, because these two fractions are both the same as 1/2 and the cross-multiplication is the same. See below ...

$$1 \cdot 6 = 6 \qquad \frac{1}{2} \stackrel{?}{=} \frac{3}{6} \qquad 2 \cdot 3 = 6$$

$$1 \cdot 6 \stackrel{?}{=} 2 \cdot 3$$
$$6 = 6.$$

To solve equations in proportion form (one fraction equals another fraction), then we cross-multiply and solve for the variable. For example ...

$$\frac{3}{x} = \frac{6}{4}$$

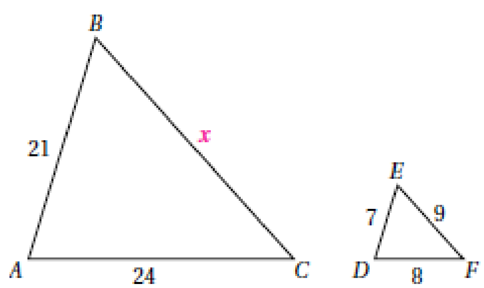
$$3 \cdot 4 = x \cdot 6 \qquad \text{Equating cross products; we could also write } 6 \cdot x = 3 \cdot 4.$$

$$\frac{12}{6} = \frac{6x}{6} \qquad \text{Dividing both sides by 6}$$

$$2 = x. \qquad \text{Simplifying}$$

There are many applications for PROPORTIONS. Be sure to review Section 7.4 for examples. One of the most common applications is Geometry. For example, similar triangles have sides that are in proportion and we can make the ratios of similar sides the same. Once again, there are many more examples in the textbook in Section 7.5.

Here are similar triangles and possible ratios that can be used to find X.



$$\frac{x}{9} = \frac{24}{8} \quad \text{and} \quad \frac{x}{9} = \frac{21}{7}.$$

MML Module #9

Chapter 8 Percent Notation

Percents can be written as decimals. For example, 10% is the same as 10/100 = 0.10 and 360% is the same as 360/100 = 3.6. To go from a percent to a decimal, move the decimal point for the percent number TWO places to the LEFT. For example,

2.15% is the same as 0.0215, or in fraction form is the same as 2.15 / 100 = 215 / 10000.

Now, to convert a decimal to a percent, move the decimal point TWO places to the RIGHT. For example,

0.16 is the same as 16.0% or 16%. In fraction form that is 16/100 = 8/50 = 4/25.

Now lets do some word problems. Use the formula A=PB, which is A is P percent of B.

HINT ... Notice how the word "IS" can be the same as the EQUAL sign and the word "OF" is the same as Multiplication. Also, the value for "P" in percent should be a fraction or decimal equivalent.

Example: Is 60% of 25 equal to 15 ?

A = P*B .... 15 = 60% * 25
15 = 0.60 * 25 = 15
Yes ... it works

Example: 32% of what number is 51.2

So we are solving for B, where A=51.2 and P=32%=0.32
51.2 = 0.32*B .. divide both sides by 32
51.2 / 0.32 = B, then B = 160

Example: 10 is what percent of 20 ?

10 = x% * 20 ... where x% is the same as x/100
10 = x/100 * 20 = 20x / 100
Lets multiply both sides by 100 to get rid of the fraction
10 * 100 = 20x
1000 = 20x .. now divide both sides by 20
x = 50 ... so the answer is 50%.

Another application of percents is the calculation of **discount rates and prices** of items. Surely we all need to know this for good shopping bargains .... and the Final Exam.

Discount = (Rate of Discount) * (Original or Marked Price)

Sale Price = (Original or Marked Price) - (Discount)

Example Original or Marked Price = $20.40
Discount Rate = 25%
Find the DISCOUNT and the SALE PRICE

DISCOUNT = (25%) * ($20.40) = 0.25 * (20.40) = $ 5.10
SALE PRICE = $20.40 - $5.10 = $15.30

MML Module #10

Chapter 9 Sets and Algebra

A **Set** is a collection of **Elements**. We often write sets using braces or brackets. For example ...

Set A = { 1,2,4,6 } ... where the elements are 1,2,4,6

Other set definitions are:
Finite Set = has a limited number of elements
Infinite Set = unlimited number of elements
Null Set { } = $\varnothing$ = no terms in the set
|A| = number of terms in set “A”, or the “**Cardinality**”
U = **Universal Set**, the total set of data for a problem

A **Subset** is a set that is part of a larger set.

Example: How many subsets can be made from (1,2,3) ?
There are a total of 8 subsets: (), (1), (2), (3), (1,2), (2,3), (1,3), and (1,2,3).
The total number of possible subsets is 2^n for "n" number of elements in a set.

Example True or False: 5 is a SUBSET of {2,4,6}
False, because 5 is not an element of the set (2,4,6)

<u>Union</u> of two sets is the combination of the elements between the sets. The notation is: $A \cup B$
Notice how the letter "U" is the same as the first the letter in the word "UNION".
Many books refer to "union" of two sets as an "or" operation.

<u>Intersection</u> of two sets are the common elements between the sets. The notation is: $A \cap B$
Notice how the inverted "U" looks like the letter "n" in the word "and".
Many books refer to "intersection" of two sets as an "and" operation

<u>Disjoint Sets</u> have no elements in common.

<u>Complement</u> of two sets is the inverse of that set, such that it does not contain any elements in the given set, but contains elements of other sets in the complete Universal Set. Recall, the Universal Set is the total set of data for a problem. The notation for complement is a "prime" symbol, such as: A'

Example: Given: A = (1,2,3), B = (2,3,4)

$U = (1,2,3,4) \ldots universal\, set$

$A \cup B = (1,2,3,4) \ldots union$

$A \cap B = (1,2) \ldots int\, er\, sec\, tion$

$A' = (4),\ B' = (1)$

$(A \cup B) = ()\ and\ (A \cap B) = (2,3)$

Example Given: U=(1,2,3,4,5,6,7,8), A=(1,2,4,5,8), B=(2,3,4,6)

Find: $A' \cup (A \cap B)$

Solution: work inside the parenthesis first to get, $(A \cap B) = (2,4)$

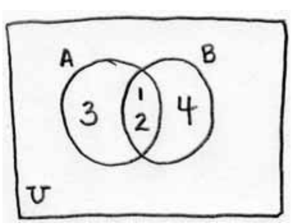
A'=(3,6,7), then $A' \cup (A \cap B) = (3,6,7) \cup (2,4) = (2,3,4,6,7)$

Here is a set theorem, where n(A) is the number of elements in a set: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
This is called the "**Cardinality Principle for Two Sets**".

A **Venn Diagram** is a pictorial representation of a set of combination of sets where the elements are contained with circles. Notice in the example below how the numbers 1 and 2 are common to both sets and therefore contained within the "A' circle and the "B" circle.

A = (1,2,3)

B = (1,2,4)



$U = \{1,2,3,4\}$

$A \cup B = \{1,2,3,4\}$

$A \cap B = \{1,2\}$

$A' = \{4\}$

$B' = \{3\}$

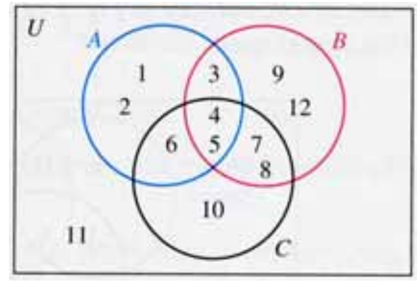
$(A \cup B)' = \varnothing$

$(A \cap B)' = \{3,4\}$

Here are some examples from this Venn Diagram.

A Venn diagram for three sets works the same as that for two sets, but there are more overlap or common areas. Sets may be equal if all elements are equal.

DeMorgan's Law: $(A \cup B)' = (A' \cap B')$ *and* $(A \cap B)' = (A' \cup B')$



Example Given the figure to the right, find: $A \cap C$

Answer = (4,5,6) … intersection or common terms

It is possible to use Venn diagrams to represent real applications of sets. Here is an example. Try to determine the full set and the subsets when making the Venn diagrams.

Example 150 students were surveyed and it was determined that 32 participated in clubs, 27 participated in sports and 18 participated in both clubs and sports. Draw a Venn diagram to find the following answers.

(a) How many participated only in clubs ?
Answer = 32-18=14
(b) How many participated only in sports ?
Answer = 27-18=9
(c) How many did not participate in either clubs or sports ?
Answer = 150-14-18-9 = 109