Evaluating Confidence Intervals for Normal (z-score) and Student's-T (t-score) Distributions

When do we use a Normal Distribution? A Normal distribution is only used when we know the standard deviation of the population or when our sample set is very large, such as more than 30.

When do we use a Student's-T Distribution? A Student's-T distribution is used when we DO NOT know the standard deviation of the population or when our sample size is small, such as less than 30.

How do we find a confidence interval? When we are evaluating any confidence interval we have to start with the expected outcome (often the mean) and then find the "error bound" which is the expected deviation from this outcome. The "error bound" is calculated using a z-score or a t-score, depending on whether we know the population standard deviation and the size of our sample set of data. We will multiply this z-score or t-score times our "standard error" to find the "error bound" for our problem. The "standard error" is typically the standard deviation of the sample set divided by the square root of the number of samples. The equation differs slightly for cases where we use population proportion

Standard Error: (Standard Deviation of Sample Set) / sqrt (samples) = $s/\sqrt{n}$

Error Bound = (Z-score or T-Score) * (Standard Error)

Confidence Interval = Mean +/- (Error Bound)

You will use this approach for the week-5 homework problems in Lane, Chapter 10 (12, 15, 18) and Illowsky, Chapter 8 (106, 112, 116, 120, 130).

What is the z-score or t-score for a given confidence interval? Please refer to my "EXCEL Tips" file in the Content-Course Resources area of our class. There is a detailed explanation of how to use EXCEL to find these values. You can also find many online calculators to get the same results. Below, I have summarized some values you will need to solve many of your homework and test problems. To find the z-score, we do not need to know the number of samples (since we assume they are large) and we just use the EXCEL …

NORMSINV (1 – (100 - Confidence Interval)/200)

Example: 95% confidence …
NORMSINV (1 – (100-95)/200) =
NORMSINV (0.975) = 1.960

Confidence Interval	Z-Score
99%	2.576
95%	1.960
90%	1.645

Now, lets discuss how we find equivalent values for a Student's-T distribution. For this case, we have to consider the number of samples in our dataset and find the "degrees of freedom", or one less than the number of samples. So, if we have 10 samples, our "degrees of freedom", or dof=9. We can then use an EXCEL formula called TINV to find the value.

TINV ((1-(Confidence Level))/100, dof)

Example: 95% confidence with 10 samples … TINV ((1-(95))/100,9) = TINV (0.05,9) = 2.262

Notice that the values of the z-score are lower than the t-score for the same confidence interval. That is because a z-score is typically reflective of a large population, so there is less deviation on the distribution and it will make your error bounds smaller. You see what happens when you put very large numbers in this formula for the sample size … such as dof=999. The answers will approach a normal distribution or z-score.

TINV (0.05,999) = 1.962, which is very close to the z-score for 95% confidence, which is 1.960.

Below is a table of t-scores for some common "degrees of freedom" values that you may find in our homework and tests.

	Confidence Interval %		
dof	**99%**	**95%**	**90%**
1	63.657	12.706	6.314
2	9.925	4.303	2.920
3	5.841	3.182	2.353
4	4.604	2.776	2.132
5	4.032	2.571	2.015
6	3.707	2.447	1.943
7	3.499	2.365	1.895
8	3.355	2.306	1.860
9	3.250	2.262	1.833
10	3.169	2.228	1.812
11	3.106	2.201	1.796
12	3.055	2.179	1.782
13	3.012	2.160	1.771
14	2.977	2.145	1.761
15	2.947	2.131	1.753
16	2.921	2.120	1.746
17	2.898	2.110	1.740
18	2.878	2.101	1.734
19	2.861	2.093	1.729
20	2.845	2.086	1.725
21	2.831	2.080	1.721
22	2.819	2.074	1.717
23	2.807	2.069	1.714
24	2.797	2.064	1.711
25	2.787	2.060	1.708
50	2.678	2.009	1.676
80	2.639	1.990	1.664
83	2.636	1.989	1.663
100	2.626	1.984	1.660
~Infinity	**2.576**	**1.960**	**1.645**